

# SUPERGRAVITY MODELS

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## ABSTRACT

Theoretical and experimental motivations behind supergravity grand unified models are described. The basic ideas of supergravity, and the origin of the soft breaking terms are reviewed. Effects of GUT thresholds and predictions arising from models possessing proton decay are discussed. Speculations as to which aspects of the Standard Model might be explained by supergravity models and which may require Planck scale physics to understand are mentioned.

## 1. INTRODUCTION

Supergravity has become the main vehicle for efforts to construct grand unified models. There are now both experimental and theoretical reasons for examining the consequences of such models. On the experimental side, there is the well known fact that measurements of  $\alpha_1 \equiv (5/3) \alpha_Y$ ,  $\alpha_2$  and  $\alpha_3$  at mass scale  $Q = M_Z$  (where  $\alpha_Y$  is the hypercharge coupling constant) allows a test of whether these three couplings of the Standard Model unify at some high scale  $Q = M_G$ . What is found [1] is that unification does not occur with the Standard Model (SM) mass spectrum but unification does appear to occur with the supersymmetrized Standard Model with one pair of Higgs doublets. Thus using the two loop renormalization group equations and making the approximation of neglecting both mass splitting of the supersymmetry (SUSY) spectrum and mass splitting of the GUT mass spectrum, one finds that

$$M_G = 10^{16.19 \pm 0.34} \text{ GeV}; \quad M_S = 10^{2.37 \pm 1.0} \text{ GeV}$$

$$\alpha_G^{-1} = 25.4 \pm 1.7 \tag{1}$$

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where  $M_S$  is the common SUSY mass, and  $\alpha_G$  is the gauge coupling constant at the unification scale  $M_G$ . (The errors in Eq. (1) are due to the errors in  $\alpha_3(M_Z)$  and we use  $\alpha_3(M_Z) = 0.118 \pm 0.007$  [2].)

There are several points worth noting about the above result:

- (i) Unification occurs only for the choice  $\alpha_1 \equiv (5/3)\alpha_Y$ , which states the way in which the hypercharge is embedded into the GUT group  $G$ . Thus unification is not completely a property of the low energy particle spectrum, but depends also on the nature of the high energy group  $G$ .
- (ii) Unification is indeed obtained by adjusting the parameter  $M_S$ . However, the significant point is that  $M_G$  and  $M_S$  come out at values that are physically acceptable, i.e.  $M_G$  is sufficiently large to inhibit proton decay, and  $M_S$  is in the correct mass region for the SUSY particles to solve the gauge hierarchy problem discussed below. (Thus  $M_S \cong 10^{2.5} \cong 300$  GeV.)
- (iii) Acceptable unification occurs only with one pair of Higgs doublets. With more Higgs doublets,  $M_G$  is so small that proton decay would already have been observed, and  $M_S$  is so large that the hierarchy problem remains.

Of course, we have no real knowledge of what the particle spectrum is above the electroweak scale. There may be additional particles at higher energies which delay or prevent grand unification from occurring. However, the simplest and most natural implication of the above result is that grand unification occurs at scale  $M_G$ , and the particle spectrum between  $M_Z$  and  $M_G$  is the supersymmetrized Standard Model with one pair of Higgs doublets.

There are also several theoretical arguments supporting the building of supersymmetric particle models. From the high energy side, string theory implies the validity of  $N = 1$  supergravity as an effective field theory below the Planck scale,  $M_{Pl} = (1/8\pi G_N)^{1/2}$  where  $G_N$  is the Newtonian constant ( $M_{Pl} = 2.4 \times 10^{18}$  GeV). Note, however, that  $M_G/M_{Pl} \approx 10^{-2}$  and so the GUT theory is moderately isolated from Planck scale physics. However, we do not expect it to be a precisely accurate theory as it may possess (1-10) % corrections from “Planck slop” terms (non-renormalizable terms scaled by powers of

$1/M_{Pl}$ ).

From the low energy electroweak scale, supersymmetry offers a solution to the well-known gauge hierarchy problem. Thus in the Standard Model, the loop corrections to the Higgs mass  $m_H$  (Fig.1) is quadratically divergent:

$$m_H^2 = m_0^2 + c(\tilde{\alpha}/4\pi)\Lambda^2 \quad (2)$$

where  $m_0$  is the bare mass,  $\tilde{\alpha}$  is a coupling constant,  $c$  is a numeric and  $\Lambda$  is the cut-off. If one takes the bare Lagrangian as fundamental, then the existence of the divergence implies that the theory is valid at energies below  $\Lambda$ , and  $\Lambda$  is the scale of new physics which intervenes to converge the integral. How large can  $\Lambda$  be, i.e. at what scale does new physics enter? Now  $m_H$  sets the electroweak scale. However, as  $\Lambda$  gets large,  $m_H$  and eventually other particle masses all get close to the large scale  $\Lambda$ . This is the “gauge hierarchy” problem which states that it is not possible to maintain a hierarchy of masses, some small at the electroweak scale and some large (e.g. at  $M_G$  or  $M_{Pl}$  scale). An alternate way of thinking of this problem is to try to choose  $m_0^2$  to cancel the large  $\Lambda^2$  term. However, for  $\Lambda \approx M_G \approx 10^{15}$  GeV, this requires fine tuning  $m_0^2$  to 24 decimal places (!) and trouble begins already for  $\Lambda \gtrsim 1$  TeV. This alternate view of the problem is known as the “fine tuning” problem. Of course, the same difficulties enter with the other divergences of relativistic quantum field theory. However, these only grow logarithmically with  $\Lambda$ , and so hierarchy difficulties only set in at the Planck scale where we already know new physics must occur.

Fig. 1 One loop correction to Higgs self mass from Higgs coupling to quarks.

Solutions to the gauge hierarchy problem fall into two categories: either one assumes the Higgs is composite (e.g. as in technicolor or  $t\bar{t}$  condensate models) and hence dissociates at scale  $\Lambda$ , or one assumes a symmetry exists to cancel the quadratic divergences. The latter possibility is supersymmetry where the Bose-Fermi symmetry causes this cancellation. For perfect supersymmetry, the two diagrams of Fig. 2 precisely cancel. If supersymmetry is broken by lifting the squark-quark degeneracy then the quadratic divergence still cancels leaving an underlying logarithmic divergence:

$$\Lambda^2 \rightarrow (m_{\tilde{q}}^2 - m_q^2) \ln(\Lambda^2/m_{\tilde{q}}^2) \quad (3)$$

Thus to avoid fine tuning we need  $m_{\tilde{q}} \gtrsim 1$  TeV, i.e.  $M_S \lesssim 1$  TeV and the new SUSY particles lie within the range for detection by current and planned accelerators. In fact, for a wide class of models it has been shown that  $m_h \lesssim 146$  GeV [3] (and usually  $m_h \lesssim 120$  GeV) which would make the light Higgs accessible to LEP200 or its upgrades.

Fig. 2. Higgs one loop corrections in supersymmetric models.  $\tilde{q}$  are spin zero squarks.

## 2. TRIVIALITY BOUND: AN ALTERNATE VIEW

The analysis given above takes the viewpoint that the bare Lagrangian is the fundamental quantity. However, the Standard Model is a renormalizable field theory. One can therefore pre-renormalize it (by introducing counter terms) and deal only with finite renormalized Green's functions. Masses and coupling constants can then be defined by these Green's functions at fixed momenta e.g. for the renormalized Higgs propagator  $\Delta_H^{(R)}(q^2)$  one may define the Higgs mass parameter  $m_H$  by  $m_H^2 = [\Delta_H^{(R)}(0)]^{-1}$ . The  $Z_2$  rescaling of  $\Delta_H^{(R)}$  can be defined by the condition  $[\partial(\Delta_H^{(R)})^{-1}/\partial q^2]_{q^2=0} = 1$ . Similarly, the  $\lambda\phi^4$

coupling constant may be defined from the renormalized 4-point vertex  $\Gamma_4^{(R)}(p_1, p_2, p_3)$  by  $\lambda = \Gamma_4^{(R)}(0, 0, 0)$ .

In the tree approximation, one has  $V_H = -m^2\phi^+\phi + \lambda(\phi^+\phi)^2$  with  $m^2, \lambda > 0$ , and defining  $\langle\phi\rangle \equiv v/\sqrt{2}$  one finds  $\langle\phi\rangle^2 = m^2/2\lambda$  and the Higgs mass to be  $m_H^2 = 2m^2$ . Since  $M_W = g_2 v/2$  (and hence  $v \cong 247$  GeV) one may write

$$M_W = \frac{g_2}{2\sqrt{2}\lambda} m_H \quad (4)$$

showing that the Higgs mass scales electroweak physics, and also that

$$\lambda = \frac{g_2^2}{8} \frac{m_H^2}{M_W^2} \quad (5)$$

If one takes now the alternate viewpoint that the renormalized field theory is the fundamental theory, one never sees a quadratic divergence (or any other divergence). Thus the theory has no problems unless it is internally inconsistent (under which circumstances it would self-destruct). This actually happens, as the theory develops a Landau pole. Letting  $\lambda(Q)$  be the running coupling constant, one finds, in the approximation of keeping only the Higgs self-couplings of  $V_H$ , the result

$$\lambda(Q) = \frac{\lambda(M_W)}{1 - \frac{3\lambda(M_W)}{4\pi^2} \ln(Q^2/M_W^2)} \quad (6)$$

where  $\lambda(M_W)$  is the low energy value given approximately by Eq. (5). A pole occurs in Eq. (6) at scale  $Q_0$  where the denominator vanishes. The theory breaks down at  $Q \approx Q_0$  and so  $Q_0$  must be a scale where new physics sets in. Using Eq. (5) one finds for this scale

$$\frac{3}{4\pi} \alpha_2 \frac{m_H^2}{M_W^2} \ln(Q_0/M_W) = 1 \quad (7)$$

In this viewpoint, the scale of new physics is determined by the *experimental* value of the Higgs mass, and the lighter the Higgs mass the larger

$Q_0$  is. For example, if  $m_H = 146$  GeV one finds  $Q_0 \cong M_{Pl}$  while if  $m_H = 500$  GeV then  $Q_0 \cong 2$  TeV. Thus, if the Higgs is light, the Standard Model could hold all the way up to the Planck scale. If the Higgs is heavy, the Standard Model must break down in the

TeV range implying an upper limit on  $m_H$ . (Of course, the argument does not exclude new physics from arising before  $Q_0$  from some other cause, but only that  $Q_0$  is an upper bound on the validity of the SM.)

The analysis given here can be extended to include gauge and Yukawa couplings, and has been performed using lattice gauge theory (as the theory becomes non-perturbative near the Landau pole). The above results remain qualitatively correct. (See, e.g. Ref [4].) Which viewpoint, the previous discussion of the gauge hierarchy problem or the Landau pole problem, determines the scale where new physics must arise depends on whether one believes the bare or renormalized theory is fundamental. In this discussion we take the gauge hierarchy problem as fundamental, and discuss the consequences of the supersymmetric solution to this difficulty.

### 3. SUSY BASICS

In supersymmetry, multiplets must have an equal number of Fermi and Bose helicity states. To build a supersymmetrized Standard Model, one needs two types of massless multiplets, chiral multiplets and vector multiplets.

Chiral Multiplets:  $(z(x), \chi(x))$

Here  $z(x)$  is a complex scalar field ( $s=0$ ) and  $\chi(x)$  is a left-handed (L) Weyl spinor ( $s = 1/2$ ). Thus  $\chi(x)$  can be used to represent quarks and leptons and also the spin 1/2 Higgsino partners of the Higgs boson, while the  $z(x)$  can be used to represent the Higgs boson and the spin 0 squarks and slepton partners of the quarks and leptons

Vector Multiplets:  $(V^\mu(x), \lambda(x))$

Here the  $V^\mu(x)$  are real vector fields ( $s = 1$ ) representing the gauge bosons, and  $\lambda(x)$  are Majorana spinors ( $s = 1/2$ ) representing the gaugino partners.

The Higgs doublets must come in pairs in supersymmetry to cancel anomalies. The minimal number is just two:

$$H_1 = (H_1^0, H_1^-); H_2 = (H_2^+, H_2^0) \quad (8)$$

The dynamics of global supersymmetry consists of gauge interactions (supersymmetrized)

and Yukawa interactions governed by the superpotential  $W$ . In general,  $W(z_a)$  is a holomorphic function of the scalar fields  $z_a$  and hence independent of the  $z_a^\dagger$ . For renormalizable interactions,  $W$  is at most cubic in the fields. Thus the most general renormalizable  $SU(3)_C \times SU(2)_L \times U(1)_Y$  and  $R$  parity invariant form for  $W$  is

$$W = \mu H_1^\alpha H_{2\alpha} + [\lambda_{ij}^{(u)} q_i^\alpha H_{2\alpha} u_j^C + \lambda_{ij}^{(d)} q_i^\alpha H_{1\alpha} d_j^C + \lambda_{ij}^{(\ell)} \ell_i^\alpha H_{1\alpha} e_j^C] \quad (9)$$

Here  $i, j = 1, 2, 3$  are generation indices,  $\alpha = 1, 2$  is the  $SU(2)_L$  index ( $H_\alpha = \varepsilon_{\alpha\beta} H^\beta$ ,  $\varepsilon_{\alpha\beta} = -\varepsilon_{\beta\alpha}$ ,  $\varepsilon_{12} = +1$ ),  $C$  = charge conjugate,  $\lambda_{ij}^{(u,d,\ell)}$  are Yukawa coupling constants and  $\mu$  is a mass scaling the Higgs mixing term. Note that the gauge invariant  $u$ -quark interaction requires the  $H_{2\alpha}$  Higgs doublet to appear, since  $H_{1\alpha}^\dagger$  cannot enter as  $W$  is holomorphic. Thus the existence of two Higgs doublets is also necessary to obtain mass growth of both the up and down quarks.

The supersymmetry invariant dynamics can be described by an effective potential

$$V = \sum_a \left| \frac{\partial W}{\partial Z_a} \right|^2 + V_D; \quad V_D = \frac{1}{2} g_i^2 D_{ir} D_{ir} \quad (10)$$

(where  $g_i$  are the  $SU(3) \times SU(2) \times U(1)$  coupling constants,  $D_{ir} = z_a^\dagger (T^{ir})_{ab} z_b$ ,  $T_{ab}^{ir}$  = group generators), and fermionic interactions

$$\mathcal{L}_Y = -\frac{1}{2} \sum_{a,b} (\bar{\chi}^{aC} \frac{\partial^2 W}{\partial z_a \partial z_b} \chi^b + h.c.) \quad (11)$$

and

$$\mathcal{L}_\lambda = -i\sqrt{2} \sum g_i \bar{\lambda}^{ir} z_a^\dagger (T^{ir})_b^a \chi_b + h.c. \quad (12)$$

Note that  $V_D$  plays the role of the  $\lambda(\phi^\dagger \phi)^2$  term in the SM, but with  $\lambda$  replaced by the gauge coupling constants  $g_i$ . It is this that allows SUSY predictions of Higgs mass bounds since the  $g_i$  are known.

After  $SU(2) \times U(1)$  breaking, the Higgsinos and  $SU(2) \times U(1)$  gauginos mix. There result 32 new SUSY particles: (i) 12 squarks ( $s=0$ , complex):  $\tilde{q}_i = (\tilde{u}_{iL}, \tilde{d}_{iL}); \tilde{u}_{iR}, \tilde{d}_{iR}$ ; (ii) 9 sleptons ( $s=0$ , complex)  $\tilde{\ell}_i = (\tilde{\nu}_{iL}, \tilde{e}_{iL}); \tilde{e}_{iR}$ ; (iii) 1 gluino ( $s = \frac{1}{2}$ , Majorana)  $\lambda^a$ ,

$a = 1 \dots 8 = SU(3)_C$  index; (iv) 2 Winos (Charginos) ( $s = \frac{1}{2}$ , Dirac).  $\tilde{W}_i$ ,  $i = 1, 2$ ,  $m_i < m_j$  for  $i < j$ ; (v) 4 Zinos (Neutralinos) ( $s = 1/2$ , Majorana)  $\tilde{Z}_i$ ,  $i = 1 \dots 4$ ,  $m_i < m_j$  for  $i < j$ ; and (vi) 4 Higgs ( $s = 0$ )  $h^0$ ,  $H^0$  real CP even;  $A^0$  real CP odd;  $H^\pm$  charged.

The  $h^0$  is the particle which most resembles the SM Higgs.

#### 4. SUPERGRAVITY BASICS

The global SUSY models discussed in the previous section possesses one serious drawback: it is not possible to achieve a phenomenologically satisfactory spontaneous breaking of supersymmetry. There are a number of reasons for this. Most obvious is that the breaking of a global symmetry implies the existence of a massless Goldstone particle, in this case a spin 1/2 particle (the Goldstino), and no candidate exists experimentally. (The neutrino interactions do not obey the correct threshold theorems.[9a]) An obvious solution to this difficulty is to promote supersymmetry to a local symmetry. The gauge particle is then spin 3/2 (the gravitino) and upon breaking of supersymmetry it absorbs the spin 1/2 Goldstino to become massive. However, supersymmetry requires that the gravitino be embedded in a massless multiplet ( $g_{\mu\nu}(x)$ ;  $\psi_\mu(x)$ ). Here  $g_{\mu\nu}(x)$  is a massless spin 2 field i.e. one is led to supergravity theory [5] where gravity is automatically included.

The coupling of supergravity to chiral and vector matter multiplets depends upon the following functions [6-9]: the superpotential  $W(z_a)$ , the Kähler potential  $d(z_z, z_a^\dagger)$ , and the gauge kinetic function  $f_{\alpha\beta}(z_a, z_a^\dagger)$  where  $\alpha, \beta$  are gauge indices. Actually,  $W$  and  $d$  enter only in the combination  $\mathcal{G} = -\kappa^2 d - \ell n [\kappa^6 W W^\dagger]$  where  $\kappa \equiv 1/M_{Pl}$ . We will assume in the following that  $d$  and  $f_{\alpha\beta}$  can be expanded in powers of fields with the higher non-linear terms scaled by  $\kappa$ :

$$d(z_a, z_a^\dagger) = c_b^a z_a z_b^\dagger + (a^{ab} z_a z_b + h.c.) + \kappa c_c^{ab} z_a z_b z_c^\dagger + \dots \quad (13)$$

$$f_{\alpha\beta}(z_a z_a^\dagger) = c_{\alpha\beta} + \kappa (c_{\alpha\beta}^a z_a + h.c.) + \dots \quad (14)$$

Supersymmetry breaking can occur at the tree level [10] or via condensates [11] due to supergravity interactions. The simplest example is to choose  $W = m^2(z + B)$ ,  $d = z_a z_a^\dagger$



and minimizing the effective potential one finds  $\langle z \rangle = \pm \kappa^{-1}(\sqrt{2} - \sqrt{6}) = O(M_{P\ell})$ . (One may further chose B to fine tune the cosmological constant to zero.) The quantity  $M_S = O(\langle \kappa^2 W \rangle) \sim \kappa m^2$  will turn out to scale the SUSY masses.

The full supergravity dynamics is quite complicated. (For a discussion see Refs. [8,12].) We list here some of the important terms. The effective potential is given by

$$V = e^{\kappa d} [(g^{-1})_b^a (\frac{\partial W}{\partial z_a} + \kappa^2 d_a W) (\frac{\partial W}{\partial z_b} + \kappa^2 d_b W)^\dagger - 3\kappa^2 |W|^2] + V_D \quad (15)$$

where

$$V_D = \frac{1}{2} g^2 \text{Re}(f^{-1})_{\alpha\beta} D_\alpha D_\beta; \quad D_\alpha = d^a (T^\alpha)_{ab} z_b \quad (16)$$

$d_a = \partial d / \partial z_a^\dagger$ ,  $d^a = \partial d / \partial z_a$  and  $g_b^a = \partial^2 d / \partial z_b \partial z_a^\dagger$ , and  $\alpha, \beta$  are gauge indices. Thus there are  $\kappa = 1/M_{P\ell}$  corrections to Eq. (10). The scalar field kinetic energy is  $-\frac{1}{2} g_b^a (D_\mu z_b) (D_\mu z_a)^\dagger$  (where  $D_\mu$  is the covariant derivative). From Eq.(13),  $g_b^a = c_b^a + O(\kappa)$  and so diagonalizing  $c_b^a$  brings the scalar kinetic energy into canonical form. The  $O(\kappa)$  correction are non-renormalizable corrections scaled by  $1/M_{P\ell}$ , and presumably small below the GUT scale. The gauge field kinetic energy is  $-\frac{1}{4} (\text{Re } f_{\alpha\beta}) F_{\mu\nu}^\alpha F^{\mu\nu\beta}$  and from Eq.(14) one sees that one obtains the canonical kinetic energy plus possible  $1/M_{P\ell}$  corrections. Finally, there is a gaugino term

$$[e^{\kappa^2 d} \kappa^2 |W| (g^{-1})_b^a d^b f_{\alpha\beta a}^\dagger] \bar{\lambda}^\alpha \lambda^\beta \quad (17)$$

where  $f_{\alpha\beta a} \equiv \partial f_{\alpha\beta} / \partial z_a$ .

To see the effects of supersymmetry breaking we consider the simple tree model discussed above where  $W = m^2(z + B)$  and  $d = z_a z_a^\dagger$ . From Eq. (15) one has the term

$$(\kappa^2 d_a W) (\kappa^2 d_a W)^\dagger \rightarrow (\kappa^2 \langle W \rangle)^2 z_a z_a^\dagger \quad (18)$$

Thus each scalar field grows a universal mass  $m_0^2 = (\kappa^2 \langle W \rangle)^2 = O(M_S^2)$ . From Eqs. (17) and (14) for the case  $z_a = z$ , a universal gaugino mass,  $m_{1/2}$ , forms of size

$$\langle \kappa^2 |W| (\partial d / \partial z) f_{\alpha\beta z}^\dagger \rangle = \langle \kappa^2 |W| z^\dagger \kappa c_{\alpha\beta}^z \rangle \quad (19)$$

and so  $m_{1/2} = O(M_S)$ . Further, by transforming the second term of Eq. (13) from the Kähler potential to the superpotential by a Kähler transformation, a Higgs mixing parameter  $\mu_0$  forms where

$$\mu_0 = \langle \kappa^2 W \rangle a^{H_1 H_2} = O(M_S) \quad (20)$$

Finally, two additional supersymmetry breaking structures arise from Eq. (15) when the matter parts of the superpotential are included:

$$A_0 W^{(3)} + B_0 W^{(2)} \quad (21)$$

where  $W^{(2,3)}$  are the (quadratic, cubic) parts of the matter superpotential.

One finds here also that

$$A_0, B_0 = O(M_S) \quad (22)$$

so that supersymmetry breaking gives rise to four soft breaking terms scaled by  $m_0$ ,  $m_{1/2}$ ,  $A_0$ ,  $B_0$  and a supersymmetric Higgs mixing parameter  $\mu_0$ . All these parameters are  $O(M_S)$ .

## 5. RADIATIVE BREAKING

A remarkable feature of supergravity GUT models is that they offer a natural explanation of  $SU(2) \times U(1)$  breaking via radiative corrections [13]. In the Standard Model,  $SU(2) \times U(1)$  breaking is *accommodated* by the device of having a negative  $(mass)^2$  for the Higgs. However, no explanation is given as to why this choice should be made. We saw in Eq. (18), that supersymmetry breaking gives rise to a universal positive  $(mass)^2$ ,  $m_0^2 > 0$ , at the scale  $Q = M_G$ . One may now run the renormalization group equations (RGE) down to the electroweak scale. As shown schematically in Fig. 3,  $m_{H_2}^2$  bends downward and eventually turns negative (due to the t-quark Yukawa couplings) signaling the breaking of  $SU(2) \times U(1)$ .

Fig. 3. Schematic diagram of running masses showing that  $m_H^2$  turns negative at the electroweak scale due to the heavy top interactions.

To see the above more quantitatively, the renormalizable Higgs interactions from Eq. (15), have the form

$$V_H = m_1^2(t) |H_1|^2 + m_2^2(t) |H_2|^2 - m_3^2(t)(H_1 H_2 + h.c.) + \frac{1}{8}[g_2^2(t) + g_Y^2(t)][|H_1|^2 - |H_2|^2]^2 + \Delta V_1 \quad (23)$$

where  $\Delta V_1$  is the one loop addition, and all parameters are running with respect to the variable  $t = \ln[M_G^2/Q^2]$ . In Eq. (23), the masses are defined by  $m_i^2(t) = m_{Hi}^2(t) + \mu^2(t)$ ,  $i = 1, 2$  and  $m_3^2(t) = -B(t)\mu(t)$  subject to the boundary conditions at  $Q = M_G$  of  $m_i^2(0) = m_0^2 + \mu_0^2$ ,  $m_3^2(t) = -B_0\mu_0$ . Minimizing the effective potential,  $\partial V_H / \partial v_i = 0$ ,  $v_i \equiv \langle H_i \rangle$  one finds

$$\frac{1}{2}M_Z^2 = \frac{\mu_1^2 - \mu_2^2 \tan^2 \beta}{\tan^2 \beta - 1}; \quad \sin 2\beta = \frac{2m_3^2}{\mu_1^2 + \mu_2^2} \quad (24)$$

where  $\mu_i^2 = m_i^2 + \Sigma_i$  and  $\tan \beta \equiv v_2/v_1$ . ( $\Sigma_i$  are the one loop corrections.) The RGE allows one to express all the parameters in Eq. (24) in terms of the GUT scale constants  $m_0, m_{1/2}, A_0, B_0$  and  $\mu_0$ . One may use Eq. (24) to eliminate  $B_0$  and  $\mu_0^2$  in terms of the remaining constants and  $\tan \beta$ . Thus one can express all 32 SUSY masses in terms of the four parameters  $m_0, m_{1/2}, A_0$  and  $\tan \beta$  and the as yet undetermined top quark mass  $m_t$ . Since the sign of  $\mu_0$  is not determined there are two branches:  $\mu_0 < 0$  and  $\mu_0 > 0$ . It is interesting to ask under what conditions will a satisfactory electroweak breaking occur. Three necessary conditions are: (i) Not all the soft breaking parameters,  $m_0, m_{1/2}, A_0$ , and  $B_0$  can be zero; (ii)  $\mu_0$  must be non-zero; (iii)  $m_t$  must be large ( $m_t > \gtrsim 90$  GeV). Thus in a real sense, item (i) implies that supersymmetry breaking triggers electroweak breaking, and from (iii) the existence of electroweak breaking predicts that the top must be heavy.

## 6. SIMPLE GUT MODEL

In Sec. 1, grand unification was discussed neglecting, however, the existence of possible GUT states which would produce threshold corrections in the vicinity of  $M_G$ . In order to

see the size of these effects, we examine here a simple  $SU(5)$  model first proposed within the framework of global supersymmetry [14]. GUT physics here is characterized by the superpotential

$$W_G = \lambda_1 \left[ \frac{1}{3} \text{Tr} \Sigma^3 + \frac{1}{2} M \text{Tr} \Sigma^2 \right] + \lambda_2 H^Y [\Sigma_Y^X + 3M' \delta_Y^X] \bar{H}_X \quad (25)$$

Here  $\Sigma_Y^X (X, Y = 1 \dots 5)$  is a 24 of  $SU(5)$ , while  $H^Y$  and  $\bar{H}_X$  are a 5 and  $\bar{5}$  of  $SU(5)$ . The  $SU(2)$  doublets of  $H^X$  and  $\bar{H}_X$  are just the  $H_1$  and  $H_2$  doublets of low energy theory. They are kept light by the choice  $M = M'$ , (which we will make here) though more natural ways of keeping the Higgs doublets light exist [15]. Upon minimizing the effective potential  $\Sigma_Y^X$  grows the VEV

$$\text{diag} \langle \Sigma_Y^X \rangle = M(2, 2, 2, -3, -3) \quad (26)$$

breaking  $SU(5)$  to the SM. We have then that  $M = O(M_G)$ . The states that become superheavy are the color triplets of  $H^X$  and  $\bar{H}_X$  transforming like  $(3, 1)$  and  $(\bar{3}, 1)$  under  $SU(3)_C \times SU(2)_L$  with mass  $M_{H_3} = 5\lambda_2 M$ , massive vector multiplets, transforming as  $(3, 2)$  and  $(\bar{3}, 2)$  with mass  $M_V = 5\sqrt{2}gM$  ( $\alpha_G \equiv g^2/4\pi$ ) and the superheavy components of  $\Sigma_Y^X$  transforming as  $(8, 1)$ ,  $(1, 3)$  and  $(1, 1)$  with masses  $M_\Sigma^8 = 5\lambda_1 M/2 = M_\Sigma^3$  and  $M_\Sigma^0 = \lambda_1 M/2$ . This model has been considered previously [16] (though with inaccurate arguments).

We limit here  $\lambda_{1,2} < 2$  (so that one stays within the perturbative domain) and also require  $\lambda_{1,2} > 0.01$  (so that the superheavy spectra stay in the GUT range). When thresholds are ignored, the RGE can be used to predict a value for  $\alpha_3(M_Z)$ . With thresholds, one gets instead a correlation between  $\alpha_3(M_Z)$  and  $M_{H_3}$ . As seen in Fig. 4 [17], one obtains an upperbound of  $\alpha_3(M_Z) < 0.135$ . Since current proton decay data requires  $M_{H_3} \gtrsim 1 \times 10^{16}$  GeV, one also gets a lower bound of  $\alpha_3(M_Z) > 0.114$ . These are consistent with the current experimental bounds of  $\alpha_3(M_Z) = 0.118 \pm 0.007$ . For the  $1\sigma$  upper limit of  $\alpha_3(M_Z) = 0.125$ , one finds  $M_{H_3} < 2 \times 10^{17}$  GeV, and so  $M_{H_3}$  is always below the Planck scale [18]. Thus the model gives generally reasonable results. Measurements of the SUSY particle masses would determine  $M_S$  which corresponds in Fig. 4 to a line in between the  $M_S = 1$  TeV and  $M_S = 30$  GeV bounding lines. That, plus an accurate measurement

of  $\alpha_3(M_Z)$ , would determine a point within the quadrilateral and hence fix  $M_{H_3}$ . Thus accurate low energy measurements would allow a prediction of the proton decay rate for  $p \rightarrow \bar{\nu} K^+$ , i.e. the model can also be experimentally tested!

Fig. 4. Grand unification constraints for the GUT model of Eq. (25). Grand unification correlates  $\alpha_3(M_Z)$  with  $M_{H_3}$ . The allowed region is within the solid quadrilateral.

## 7. GUT PHYSICS OR PLANCK PHYSICS?

Supergravity GUT models do not represent a fundamental theory, but rather an effective theory valid at energies below  $M_G$ . One may ask what aspects of the theory can be understood at the GUT level, and what requires higher scale physics, presumably unknown Planck scale physics, to understand. We list here a few speculations.

- (i) Unification of gauge couplings. This is presumably GUT physics since it depends on the particle spectrum below  $M_G$  and on the grand unification group  $G$  which holds above  $M_G$ .
- (ii) Quark/lepton masses, KM matrix elements, Yukawa coupling constants are presumably Planck scale physics (e.g. as in string theory) except for possible symmetry constraints that the GUT group  $G$  may impose.
- (iii) Nature of supersymmetry breaking. The structure of the hidden sector where supersymmetry breaking takes place is presumably Planck physics. However, it can be

- parameterized at the GUT scale in terms of five parameters  $m_0$ ,  $m_{1/2}$ ,  $A_0$ ,  $B_0$  and  $\mu_0$ .
- (iv) Squark/slepton masses and widths. This is GUT physics, once the five hidden sector parameters are chosen.
  - (v) Electroweak breaking. This is GUT physics, once the hidden sector parameters are chosen.
  - (vi) Proton stability. GUT physics depending on the interactions which break  $G$  to the SM group.

We see from the above, that while supergravity grand unified models add significantly to our understanding of low energy physics, there are a number of areas, notably in the Yukawa couplings and in the structure of the hidden sector, where it offers no new insights. For these one must make a phenomenological treatment.

## 8. PROTON DECAY

There are two main modes of proton decay in GUT models:  $p \rightarrow e^+ + \pi^0$  and  $p \rightarrow \bar{\nu} + K^+$ . The former can occur in both SUSY and non-SUSY grand unification, and generally will occur for any model whose grand unification group  $G$  possesses  $SU(5)$  as a subgroup. The latter is a specifically supersymmetric mode. Thus the observation of  $p \rightarrow \bar{\nu}K^+$  would be a strong indication of the validity of supergravity grand unification. This decay can also occur when  $G$  possesses an  $SU(5)$  subgroup and if the light matter below  $M_G$  is embedded in the usual way in 10 and  $\bar{5}$  representations of the  $SU(5)$  subgroup. However, it is possible to construct a complicated Higgs sector where one fine tunes the  $p \rightarrow \bar{\nu}K$  amplitude to zero and still maintains only too light Higgs doublets below  $M_G$ . However, such models appear somewhat artificial, and the  $p \rightarrow \bar{\nu}K$  decay mode is generally expected to arise, though it can be evaded.

- (i)  $p \rightarrow e^+ \pi^0$ . This mode proceeds as in non-SUSY GUTs through the superheavy vector bosons of mass  $M_V = O(M_G)$ . For SUSY models one has [19]

$$\tau(p \rightarrow e^+ \pi^0) = 10^{31 \pm 1} \left( \frac{M_V}{6 \times 10^{14} \text{ GeV}} \right)^4 \text{yr} \quad (27)$$

The current experimental bound is [20]  $\tau(p \rightarrow e^+ \pi^0) > 5.5 \times 10^{32} \text{ yr}$  (90% CL). Super Kamiokande expects to be sensitive up to a lifetime  $\tau(p \rightarrow e^+ \pi^0) < 1 \times 10^{34} \text{ yr}$

[21]. From Eq. (27) this would require  $M_V \lesssim 6 \times 10^{15}$  GeV for the decay mode to be observable.

- (ii)  $p \rightarrow \bar{\nu} K^+$ . For the models discussed above, this mode proceeds through the exchange of the superheavy Higgsino color triplet as can be seen in Fig. 5 [22,23]. Current data [20] gives the bound  $\tau(p \rightarrow \bar{\nu} K^+) > 1 \times 10^{32}$  yr (90% CL). From Fig. 5, one sees that the amplitude for decay depends on  $1/M_{H_3}$ . The current data then puts a bound of  $M_{H_3} \gtrsim 1 \times 10^{16}$  GeV [24]. Future experiments expect an increased sensitivity for Super Kamiokande of up to  $\tau(p \rightarrow \bar{\nu} K^+) < 2 \times 10^{33}$  yr [21], and for ICARUS of up to  $\tau(p \rightarrow \bar{\nu} K^+) < 5 \times 10^{33}$  yr [25]. Thus the GUT model of Sec. 6, where  $M_{H_3} < 2M_V$ , would predict that if the  $p \rightarrow e^+ \pi^0$  mode at future experiments were observed, the  $p \rightarrow \bar{\nu} K^+$  should be seen very copiously as then  $M_{H_3}$  would be less than  $1.2 \times 10^{16}$  GeV.

Fig. 5. Example of diagram contributing to the decay  $p \rightarrow \bar{\nu}_\mu K^+$ . There are additional diagrams with  $\bar{\nu}_\tau$  and  $\bar{\nu}_e$  final state. CKM matrix elements appear at the  $\tilde{W}$  vertices.

The  $p \rightarrow \bar{\nu} K^+$  amplitude depends not only on  $M_{H_3}$  but also in a detailed way, on the SUSY masses of the particles in the loop of Fig. 5 [23]. Since as discussed in Sec. 5, these masses are functions of the basic parameters, which we may choose to be  $m_0$ ,  $m_{\tilde{g}} = [\alpha_3(m_{\tilde{g}})/\alpha_G] m_{1/2}$ ,  $A_t$  (The t-quark A parameter at the electroweak scale) and  $\tan \beta$ , the current bounds on p-decay give rise to bounds in this parameter space. If we restrict  $M_{H_3} < 2 \times 10^{17}$  GeV (which keeps  $M_{H_3}/M_{Pl} < 1/10$  and is what is implied by the GUT model of Sec. 6) one finds the restrictions  $\tan \beta l 8, |A_t/m_0| l 2$  and in most of the parameter space  $m_0 > m_{\tilde{g}}$ . Fig. 6 [26] shows what can be expected from future

proton decay experiments. Thus if we require  $m_0 \leq 1$  TeV (to prevent excessive fine tuning), we see that ICARUS should detect  $p \rightarrow \bar{\nu} K^+$  proton decay for even the largest value of  $M_{H_3}$  considered here (and Super Kamiokande should similarly detect this mode for  $m_0 \leq 950$  GeV) if  $m_{\tilde{W}_1} > 100$  GeV. Thus if these experiments do not see proton decay, then  $m_{\tilde{W}_1} < 100$  GeV, and hence the light Wino should be observable at LEP 200. In either case,  $m_{\tilde{W}_1} < 100$  GeV or  $m_{\tilde{W}_1} > 100$  GeV these models with  $SU(5)$  type proton decay imply that a signal of supersymmetry should be observed, and this could occur prior to the turning on of the LHC or SSC.

Fig. 6. Maximum value of  $\tau(p \rightarrow \bar{\nu} K^+)$  for  $m_t = 150$  GeV,  $\mu < 0$  subject to the constraint  $m_{\tilde{W}_1} > 100$  GeV. The dash-dot curve is for  $M_{H_3} = 2 \times 10^{17}$  GeV. The dashed curve for  $M_{H_3} = 1.2 \times 10^{17}$  GeV, and the solid curve for  $M_{H_3} = 6 \times 10^{16}$  GeV. The horizontal upper and lower lines are the bounds of ICARUS and Super Kamiokande.

## 9. CONCLUSIONS

Supersymmetry represents a natural way of solving the gauge hierarchy problem. Local supersymmetry, i.e. supergravity, supplies a formal structure for treating supersymmetric grand unified models which allow for a consistent treatment of spontaneous breaking of supersymmetry. The supergravity GUT models have a large amount of predictive ability in that the 32 SUSY particle masses are determined from only five parameters.



One set of mass relations which holds in several models over most of the parameter space is the following scaling relations [24, 27]:

$$2m_{\tilde{Z}_1} \cong m_{\tilde{W}_1} \cong m_{\tilde{Z}_2} \quad (28)$$

$$m_{\tilde{W}_2} \cong m_{\tilde{Z}_3} \cong m_{\tilde{Z}_4} \gg m_{\tilde{Z}_1} \quad (29)$$

$$m_{\tilde{W}_1} \simeq \frac{1}{3}m_{\tilde{g}} \text{ for } \mu < 0; m_{\tilde{W}_1} \simeq \frac{1}{4}m_{\tilde{g}} \text{ for } \mu > 0 \quad (30)$$

and

$$m_{H^0} \cong m_A \cong m_{H^\pm} \gg m_h \quad (31)$$

These relations are actually the remnants of the gauge hierarchy problem. Thus in most of the allowed parameter space one has  $m_0^2, m_{\tilde{g}}^2 \gg M_Z^2$  (which occurs already when  $m_0, m_{\tilde{g}} \gtrsim (2-3)M_Z$ ). In the radiative breaking equations, this usually means then that  $\mu^2 \gg M_Z^2$  to guarantee enough cancellation so that the r.h.s. of the first equation in Eq. (24) correctly adds up to only  $\frac{1}{2}M_Z^2$ . One can then check that Eqs. (28) - (31) are a consequence of  $\mu^2 \gg M_Z^2$  etc. A verification of Eqs. (28) - (31) would be strong support of supergravity GUT models as they depend strongly on how the structure of the theory at the GUT scale accomplishes  $SU(2) \times U(1)$  breaking at the electroweak scale.

Finally, we should like to stress that in spite of the ability of supergravity GUT models to make testable predictions such as the ones discussed above, even if it is a valid idea, it must still be viewed as an approximate effective theory holding at scales below  $M_G$ . The closeness of  $M_G$  to  $M_{Pl}$ , i.e.  $M_G/M_{Pl} \simeq 1/10 - 1/100$ , implies then that the theory may possess  $\approx (1-10)\%$  errors in its predictions, and precision experiments on the validity of these models could conceivably yield information on the nature of Planck scale physics.

## ACKNOWLEDGEMENTS

This work was supported in part by the National Science Foundation Grants Nos. PHY-916593 and PHY-93-06906. One of us (R.A.) would like to thank the Department of

Theoretical Physics, Oxford University for its kind hospitality during the writing of this report.

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